Monitoring, Sanctions and Front-Loading of Job Search in a Non-Stationary Model: Internet Appendix

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Summary

This file provides some supporting results and derivations excluded from the main text due to space limits.

A Lifetime Value of Unemployment

Here we present an alternative derivation of the lifetime value of unemployment using the continuous-time Bellman equation. Consider an interval $[t_0, t_1)$. For any $\tau \in [t_0, t_1)$ period value of unemployment is the instantaneous utility of unemployment at τ plus the expected change in the value of unemployment at τ, maximized with respect to control variables, i.e.

$$\rho U(\tau) = \max_{s(\tau), w_r(\tau)} \left\{ y_h(\tau) + \frac{1}{d\tau} E_\tau dU(\tau) \right\}. \tag{A-1}$$

Let stochastic transition to employment be governed by a Poisson process $q(\tau)$ with an arrival rate $p(\tau)$. Then stochastic differential of the value unemployment reads

$$dU(\tau) = \dot{U}(\tau)\, d\tau + \left[\bar{W}(\tau) - U(\tau)\right] dq(\tau). \tag{A-2}$$

Inserting this into (A-1) and taking expectation at τ we get

$$\rho U(\tau) = \max_{s(\tau), w_r(\tau)} \left\{ y_h(\tau) + \dot{U}(\tau) + p(\tau)\left[\bar{W}(\tau) - U(\tau)\right] \right\}.$$

At the optimal solution for control variables the maximized Bellman equation becomes

$$\rho U(\tau) = y_h(\tau) + \dot{U}(\tau) + p(\tau)\left[\bar{W}(\tau) - U(\tau)\right],$$

which can be rearranged to get

$$\dot{U}(\tau) - [\rho + p(\tau)]\, U(\tau) = -\left[y_h(\tau) + p(\tau)\, \bar{W}(\tau)\right]$$

and, multiplying both sides by $P(\tau, t_0)\, e^{-\rho(\tau - t_0)}$,

$$\dot{U}(\tau)\, P(\tau, t_0)\, e^{-\rho(\tau-t_0)} - [\rho + p(\tau)]\, U(\tau)\, P(\tau, t_0)\, e^{-\rho(\tau-t_0)} = -\left[y_h(\tau) + p(\tau)\, \bar{W}(\tau)\right] P(\tau, t_0)\, e^{-\rho(\tau-t_0)}$$

$$\Leftrightarrow \quad \frac{\partial}{\partial \tau}\left(U(\tau)\, P(\tau, t_0)\, e^{-\rho(\tau-t_0)}\right) = -\left[y_h(\tau) + p(\tau)\, \bar{W}(\tau)\right] P(\tau, t_0)\, e^{-\rho(\tau-t_0)}.$$

Integrating this expression from t_0 to t_1 we get

$$U(t_1)\, P(t_1, t_0)\, e^{-\rho(t_1-t_0)} - U(t_0) = -\int_{t_0}^{t_1} \left[y_h(\tau) + p(\tau)\, \bar{W}(\tau)\right] P(\tau, t_0)\, e^{-\rho(\tau-t_0)} d\tau. \tag{A-3}$$

Rearranged, this equation gives us equation (6) in the main text, evaluated at the optimal solution.

B Optimal Paths and Endpoint Conditions

In this section we consider optimal paths for control variables, along with the corresponding endpoint conditions, derived for the institutional setting of Section 3.2 in the main text. The exposition focuses on the time span between two adjacent interviews (interview "$k-1$" and interview "k") and explicitly considers the difference between the scheduled interval and the time of interview delay.

B.1 Scheduled Interval

Consider first the scheduled interval $[t^*_{k-1}, t'_k)$. Lifetime utility at t^*_{k-1} is given by

$$U^e_{k,1}\left(t^*_{k-1}\right) = \int_{t^*_{k-1}}^{t'_k} \left[y^e_{j(k)}(\tau) + p^e(\tau)\, \bar{W}^e_k(\tau)\right] P^e\left(\tau, t^*_{k-1}\right) e^{-\rho(\tau - t^*_{k-1})} d\tau$$
$$+ \mathbb{U}^e_{k,1}\left(t'_k\right) P^e\left(t'_k, t^*_{k-1}\right) e^{-\rho(t'_k - t^*_{k-1})} \tag{B-1}$$

$$\mathbb{U}^e_{k,1}\left(t'_k\right) = U^e_{k,2}\left(t'_k\right) \tag{B-2}$$

where $j(k) = h$ for $k \leq 2$, $j(k) = l$ for $k = 3$, and $y^e_{j(k)}(\tau) \equiv b_{j(k)} + v - c[s^e(\tau)]$. Following the steps outlined in Appendix A.2 in the main text, the generalized current value Hamiltonian at the scheduled interval reads

$$\tilde{H}^e_{k,1}(\tau) = b_{j(k)} + v - c[s^e(\tau)] + p^e(\tau)\left[\bar{W}^e_k(\tau) - U^e_{k,1}(\tau)\right] + \tilde{\lambda}^e_S(\tau)\, \frac{s^e(\tau) - \bar{S}^e\left(\tau, t^*_{k-1}\right)}{\tau - t^*_{k-1}}.$$

Using the fact that $\bar{W}^e_k(\tau) = \frac{1}{(\rho+\delta)\bar{F}(w^e_r(\tau))} \int_{w^e_r(\tau)}^{\infty} \left[w + \delta U^1_{k,1}(\tau)\right] dF(w)$ once transition to employment is anytime within $[t^*_{k-1}, t'_k)$, we can write $\tilde{H}^e_{k,1}(\tau)$ in a slightly more convenient form, namely

$$\tilde{H}^e_{k,1}(\tau) = b_{j(k)} + v - c[s^e(\tau)]$$
$$+ \frac{s^e(\tau)}{\rho+\delta} \int_{w^e_r(\tau)}^{\infty} \left\{w - \rho U^e_{k,1}(\tau) + \delta\left[U^1_{k,1}(\tau) - U^e_{k,1}(\tau)\right]\right\} dF(w) + \tilde{\lambda}^e_S(\tau)\, \frac{s^e(\tau) - \bar{S}^e\left(\tau, t^*_{k-1}\right)}{\tau - t^*_{k-1}}.$$

B.1.1 First order conditions

- *Control variables*

Considering search effort $s(\tau)$:

$$\frac{\partial \tilde{H}^e_{k,1}(\tau)}{\partial s^e(\tau)} = -c'[s^e(\tau)] + \frac{1}{\rho+\delta} \int_{w^e_r(\tau)}^{\infty} \left\{w - \rho U^e_{k,1}(\tau) + \delta\left[U^1_{k,1}(\tau) - U^e_{k,1}(\tau)\right]\right\} dF(w) + \frac{\tilde{\lambda}^e_S(\tau)}{\tau - t^*_{k-1}} = 0$$

$$\Leftrightarrow \quad c'\left[s^{e}\left(\tau\right)\right]=\frac{1}{\rho+\delta}\int_{w_{r}^{e}\left(\tau\right)}^{\infty}\left\{w-\rho U_{k,1}^{e}\left(\tau\right)+\delta\left[U_{k,1}^{1}\left(\tau\right)-U_{k,1}^{e}\left(\tau\right)\right]\right\}dF\left(w\right)+\frac{\tilde{\lambda}_{S}^{e}\left(\tau\right)}{\tau-t_{k-1}^{*}}. \tag{B-3}$$

Considering reservation wage $w_r(\tau)$:

$$\frac{\partial\tilde{H}_{k,1}^{e}\left(\tau\right)}{\partial w_{r}^{e}\left(\tau\right)}=-\frac{s^{e}\left(\tau\right)}{\rho+\delta}\left\{w_{r}^{e}\left(\tau\right)-\rho U_{k,1}^{e}\left(\tau\right)+\delta\left[U_{k,1}^{1}\left(\tau\right)-U_{k,1}^{e}\left(\tau\right)\right]\right\}f\left(w_{r}^{e}\left(\tau\right)\right)=0$$

$$\Leftrightarrow \quad w_{r}^{e}\left(\tau\right)=\rho U_{k,1}^{e}\left(\tau\right)-\delta\left[U_{k,1}^{1}\left(\tau\right)-U_{k,1}^{e}\left(\tau\right)\right]. \tag{B-4}$$

- *State variables*

For average search effort $\bar{S}^e\left(\tau, t_{k-1}^*\right)$:

$$\frac{\partial\tilde{H}_{k,1}^{e}\left(\tau\right)}{\partial\bar{S}^{e}\left(\tau,t_{k-1}^{*}\right)}=-\frac{\tilde{\lambda}_{S}^{e}\left(\tau\right)}{\tau-t_{k-1}^{*}}=\left[p^{e}\left(\tau\right)+\rho\right]\tilde{\lambda}_{S}^{e}\left(\tau\right)-\dot{\tilde{\lambda}}_{S}^{e}\left(\tau\right)$$

$$\Leftrightarrow \quad \frac{\dot{\tilde{\lambda}}_{S}^{e}\left(\tau\right)}{\tilde{\lambda}_{S}^{e}\left(\tau\right)}=p^{e}\left(\tau\right)+\rho+\frac{1}{\tau-t_{k-1}^{*}}.$$

Acknowledging that $\dot{\tilde{\lambda}}_{S}^{e}\left(\tau\right)/\tilde{\lambda}_{S}^{e}\left(\tau\right)=\frac{\partial}{\partial\tau}\ln\tilde{\lambda}_{S}^{e}\left(\tau\right)$, $p^{e}\left(\tau\right)=-\frac{\dot{P}^{e}\left(\tau,t_{k-1}^{*}\right)}{P^{e}\left(\tau,t_{k-1}^{*}\right)}=-\frac{\partial}{\partial\tau}\ln P^{e}\left(\tau,t_{k-1}^{*}\right)$ and $\frac{1}{\tau-t_{k-1}^{*}}=\frac{\partial}{\partial\tau}\ln\left(\tau-t_{k-1}^{*}\right)$, the last result can be written as

$$\frac{\partial}{\partial\tau}\ln\tilde{\lambda}_{S}^{e}\left(\tau\right)=-\frac{\partial}{\partial\tau}\ln P^{e}\left(\tau,t_{k-1}^{*}\right)+\rho+\frac{\partial}{\partial\tau}\ln\left(\tau-t_{k-1}^{*}\right).$$

Integrating from τ to t'_k and rearranging

$$\ln\frac{\tilde{\lambda}_{S}^{e}\left(t_{k}'\right)}{\tilde{\lambda}_{S}^{e}\left(\tau\right)}=-\ln P^{e}\left(t_{k}',\tau\right)+\rho\left(t_{k}'-\tau\right)+\ln\frac{t_{k}'-t_{k-1}^{*}}{\tau-t_{k-1}^{*}}$$

$$\Leftrightarrow \quad \frac{\tilde{\lambda}_{S}^{e}\left(t_{k}'\right)}{\tilde{\lambda}_{S}^{e}\left(\tau\right)}=\frac{t_{k}'-t_{k-1}^{*}}{\tau-t_{k-1}^{*}}\frac{e^{\rho\left(t_{k}'-\tau\right)}}{P^{e}\left(t_{k}',\tau\right)}.$$

Applying the transversality condition

$$\tilde{\lambda}_{S}^{e}\left(t_{k}'\right)=\frac{\partial\mathbb{U}_{k,1}^{e}\left(t_{k}'\right)}{\partial\bar{S}^{e}\left(t_{k}',t_{k-1}^{*}\right)}=\frac{\partial U_{k,2}^{e}\left(t_{k}'\right)}{\partial\bar{S}^{e}\left(t_{k}',t_{k-1}^{*}\right)}$$

we finally see that

$$\tilde{\lambda}_{S}^{e}\left(\tau\right)=\frac{\tau-t_{k-1}^{*}}{t_{k}'-t_{k-1}^{*}}\frac{\partial U_{k,2}^{e}\left(t_{k}'\right)}{\partial\bar{S}^{e}\left(t_{k}',t_{k-1}^{*}\right)}P^{e}\left(t_{k}',\tau\right)e^{-\rho\left(t_{k}'-\tau\right)}.$$

Inserting this result, together with (B-4), into (B-3) we obtain first order condition for search effort

$$c'\left[s^{e}\left(\tau\right)\right]=\frac{1}{\rho+\delta}\int_{w_{r}^{e}\left(\tau\right)}^{\infty}\left[w-w_{r}^{e}\left(\tau\right)\right]dF\left(w\right)$$
$$+\frac{1}{t_{k}'-t_{k-1}^{*}}\frac{\partial U_{k,2}^{e}\left(t_{k}'\right)}{\partial\bar{S}^{e}\left(t_{k}',t_{k-1}^{*}\right)}P^{e}\left(t_{k}',\tau\right)e^{-\rho\left(t_{k}'-\tau\right)} \tag{B-5}$$

with costate substituted out.

Equations (B-4) and (B-5) constitute the system of first order conditions for the control variables on the scheduled interval.

B.1.2 Evolution over time and endpoint conditions

Differentiating equations (B-4) and (B-5) with respect to time we get the optimal paths of the control variables at the scheduled interval. Optimal path for the reservation wage reads

$$\dot{w}_r^e(\tau) = \rho \dot{U}_{k,1}^e(\tau) - \delta[\dot{U}_{k,1}^1(\tau) - \dot{U}_{k,1}^e(\tau)]. \tag{B-6}$$

Optimal path for the search effort is likewise

$$\begin{aligned} c''[s^e(\tau)]\,\dot{s}^e(\tau) &= \frac{1}{\rho+\delta}\frac{\partial}{\partial \tau}\left(\int_{w_r^e(\tau)}^{\infty}[w - w_r^e(\tau)]\,dF(w)\right) \\ &\qquad + \frac{1}{t_k' - t_{k-1}^*}\frac{\partial U_{k,2}^e(t_k')}{\partial \bar{S}^e(t_k', t_{k-1}^*)}\frac{\partial}{\partial \tau}\left(P^e(t_k',\tau)e^{-\rho(t_k'-\tau)}\right) \\ \Leftrightarrow\quad \dot{s}^e(\tau) &= \frac{1}{(\rho+\delta)\,c''[s^e(\tau)]}\int_{w_r^e(\tau)}^{\infty}[-\dot{w}_r^e(\tau)]\,dF(w) \\ &\qquad + \frac{p^e(\tau)+\rho}{c''[s^e(\tau)]}\frac{1}{t_k' - t_{k-1}^*}\frac{\partial U_{k,2}^e(t_k')}{\partial \bar{S}^e(t_k', t_{k-1}^*)}P^e(t_k',\tau)e^{-\rho(t_k'-\tau)} \end{aligned}$$

Once $e = 1$, the last term drops out, since accumulated search effort $\bar{S}^1(t_k', t_{k-1}^*)$ does not affect the probability of a negative evaluation during the delay interval. Otherwise, when $e = 0$, the term containing the derivative multiplied by the expected discounted capital loss due to negative evaluation can be substituted out using eq.(B-5). Thus, optimal path for search effort reads

$$\dot{s}^e(\tau) = -\frac{\dot{w}_r^e(\tau)\,\bar{F}(w_r^e(\tau))}{(\rho+\delta)\,c''[s^e(\tau)]} + (1-e)\frac{p^e(\tau)+\rho}{c''[s^e(\tau)]}\left[c'[s^e(\tau)] - \frac{1}{\rho+\delta}\int_{w_r^e(\tau)}^{\infty}[w - w_r^e(\tau)]\,dF(w)\right]. \tag{B-7}$$

Both paths depend on the evolution of the value of unemployment: $\dot{w}_r^e(\tau)$ explicitly and $\dot{s}^e(\tau)$ implicitly via $\dot{w}_r^e(\tau)$. Evolution of the value of unemployment can be obtained simply by differentiating (B-1) at any $\tau \in [t_{k-1}^*, t_k')$. We get

$$\begin{aligned} \dot{U}_{k,1}^e(\tau) &= \int_{\tau}^{t_k'}\left[y_{j(k)}^e(x) + p^e(x)\,\bar{W}_k^e(x)\right]\frac{\partial}{\partial \tau}\left(P^e(x,\tau)\,e^{-\rho(x-\tau)}\right)dx - [y_{j(k)}^e(\tau) + p^e(\tau)\,\bar{W}_k^e(\tau)] \\ &\quad + \mathbb{U}_{k,1}^e(t_k')\frac{\partial}{\partial \tau}\left(P^e(t_k',\tau)\,e^{-\rho(t_k'-\tau)}\right) \\ &= (p^e(\tau)+\rho)\int_{\tau}^{t_k'}\left[y_{j(k)}^e(x) + p^e(x)\,\bar{W}^e(x)\right]P^e(x,\tau)\,e^{-\rho(x-\tau)}dx \\ &\quad + (p^e(\tau)+\rho)\,\mathbb{U}_{k,1}^e(t_k')\,P^e(t_k',\tau)\,e^{-\rho(t_k'-\tau)} - [y_{j(k)}^e(\tau) + p^e(\tau)\,\bar{W}_k^e(\tau)] \\ &= (p^e(\tau)+\rho)\,U_{k,1}^e(\tau) - [y_{j(k)}^e(\tau) + p^e(\tau)\,\bar{W}_k^e(\tau)] \\ &= \rho U_{k,1}^e(\tau) - y_{j(k)}^e(\tau) - p^e(\tau)\left[\bar{W}_k^e(\tau) - U_{k,1}^e(\tau)\right]. \end{aligned}$$

Inserting into the last expression eq.(B-4), which tells us that $U_{k,1}^e(\tau) = \frac{1}{\rho+\delta}[w_r^e(\tau) + \delta U_{k,1}^1(\tau)]$, and the expression for $\bar{W}_k^e(\tau)$ we finally get

$$\dot{U}_{k,1}^e(\tau) = \frac{\rho}{\rho+\delta}w_r^e(\tau) - b_{j(k)} - v + c[s^e(\tau)] - \frac{s^e(\tau)}{\rho+\delta}\int_{w_r^e(\tau)}^{\infty}[w - w_r^e(\tau)]\,dF(w) + \frac{\rho\delta}{\rho+\delta}U_{k,1}^1(\tau). \tag{B-8}$$

Differential equations (B-6), (B-7) and (B-8) form a system that describes the evolution of optimal controls and the lifetime utility in the scheduled interval. This system can be solved backwards from the endpoint conditions for all the paths at t'_k. Endpoint condition for the utility function at t'_k is given by $U^e_{k,1}(t'_k) = U^e_{k,2}(t'_k)$. Endpoint conditions for reservation wage and search effort are found by solving the system of F.O.C. (B-4)-(B-5) evaluated at t'_k

$$\begin{cases} w^e_r(t'_k) = \rho U^e_{k,2}(t'_k) - \delta\left[U^1_{k,2}(t'_k) - U^e_{k,2}(t'_k)\right] \\ c'\left[s^e(t'_k)\right] = \frac{1}{\rho+\delta}\int_{w^e_r(t'_k)}^{\infty}\left[w - w^e_r(t'_k)\right]dF(w) + \frac{1}{t'_k - t^*_{k-1}}\frac{\partial U^e_{k,2}(t'_k)}{\partial \bar{S}^e(t'_k, t^*_{k-1})} \end{cases}$$

for $\{w^e_r(t'_k), s^e(t'_k)\}$, where we have already invoked that $U^e_{k,1}(t'_k) = U^e_{k,2}(t'_k)$.

B.2 Delay Interval

Consider first the interval past the scheduled date, but before the arrival of the next interview $[t'_k, t^*_k)$. The realized delay t^*_k is the minimum of the random draw form the distribution of delay time with rate q and a maximum possible delay $\bar{t}^*_k$. Lifetime utility at t'_k therefore is given by

$$U^e_{k,2}(t'_k) = \int_{t'_k}^{\bar{t}^*_k}\left[y^e_{j(k)}(\tau) + p^e(\tau)\bar{W}^e_k(\tau) + q\mathbb{U}^e_{k,2}(\tau)\right]P^e(\tau, t'_k)\,e^{-[\rho+q](\tau - t'_k)}d\tau + \mathbb{U}^e_{k,2}(\bar{t}^*_k)\,P^e(\bar{t}^*_k, t'_k)\,e^{-[\rho+q](\bar{t}^*_k - t'_k)} \quad \text{(B-9)}$$

$$\mathbb{U}^e_{k,2}(\tau) = \pi^e_k\left[\bar{S}^e(\tau, t^*_{k-1})\right]U^e_{k+1,1}(t^*_k) + \left(1 - \pi^e_k\left[\bar{S}^e(\tau, t^*_{k-1})\right]\right)U^+ \quad \text{(B-10)}$$

where $j(k)$ and $y^e_{j(k)}(\tau)$ are as defined before. Following the steps outlined in Appendix A.2 in the main text, the generalized current value Hamiltonian at the delay interval reads

$$\tilde{H}^e_{k,2}(\tau) = b_{j(k)} + v - c\left[s^e(\tau)\right] + p^e(\tau)\left[\bar{W}^e_k(\tau) - U^e_{k,2}(\tau)\right] + q\mathbb{U}^e_{k,2}(\tau) + \tilde{\lambda}^e_S(\tau)\frac{s^e(\tau) - \bar{S}^e(\tau, t^*_{k-1})}{\tau - t^*_{k-1}}.$$

Using the fact that $\bar{W}^e_k(\tau) = \frac{1}{(\rho+\delta)\bar{F}(w^e_r(\tau))}\int_{w^e_r(\tau)}^{\infty}\left[w + \delta U^1_{k,2}(\tau)\right]dF(w)$ once transition to employment is anytime within $[t'_k, \bar{t}^*_k)$, we can write $\tilde{H}^e_{k,2}(\tau)$ in a slightly more convenient form, namely

$$\begin{aligned}\tilde{H}^e_{k,2}(\tau) &= b_{j(k)} + v - c\left[s^e(\tau)\right] + q\mathbb{U}^e_{k,2}(\tau) \\ &+ \frac{s^e(\tau)}{\rho+\delta}\int_{w^e_r(\tau)}^{\infty}\left\{w - \rho U^e_{k,2}(\tau) + \delta\left[U^1_{k,2}(\tau) - U^e_{k,2}(\tau)\right]\right\}dF(w) + \tilde{\lambda}^e_S(\tau)\frac{s^e(\tau) - \bar{S}^e(\tau, t^*_{k-1})}{\tau - t^*_{k-1}}.\end{aligned}$$

B.2.1 First order conditions

- *Control variables*

F.O.C. for control variables repeat the derivations in B.1.1 leading us to

$$c'\left[s^e(\tau)\right] = \frac{1}{\rho+\delta}\int_{w^e_r(\tau)}^{\infty}\left\{w - \rho U^e_{k,2}(\tau) + \delta\left[U^1_{k,2}(\tau) - U^e_{k,2}(\tau)\right]\right\}dF(w) + \frac{\tilde{\lambda}^e_S(\tau)}{\tau - t^*_{k-1}}, \quad \text{(B-11)}$$

$$w^e_r(\tau) = \rho U^e_{k,2}(\tau) - \delta\left[U^1_{k,2}(\tau) - U^e_{k,2}(\tau)\right]. \quad \text{(B-12)}$$

- *State variables*

For average search effort $\bar{S}^e(\tau, t^*_{k-1})$:

$$\frac{\partial \tilde{H}^e_{k,2}(\tau)}{\partial \bar{S}^e(\tau, t^*_{k-1})} = q\frac{\partial \pi^e_k\left[\bar{S}^e(\tau, t^*_{k-1})\right]}{\partial \bar{S}^e(\tau, t^*_{k-1})}\left[U^e_{k+1,1}(t^*_k) - U^+\right] - \frac{\tilde{\lambda}^e_S(\tau)}{\tau - t^*_{k-1}} = \left[p^e(\tau) + \rho + q\right]\tilde{\lambda}^e_S(\tau) - \dot{\tilde{\lambda}}^e_S(\tau)$$

$$\Leftrightarrow \quad \dot{\tilde{\lambda}}^e_S(\tau) - \left(p^e(\tau) + \rho + q + \frac{1}{\tau - t^*_{k-1}}\right)\tilde{\lambda}^e_S(\tau) = -q\frac{\partial \pi^e_k\left[\bar{S}^e(\tau, t^*_{k-1})\right]}{\partial \bar{S}^e(\tau, t^*_{k-1})}\left[U^e_{k+1,1}(t^*_k) - U^+\right]$$

Multiplying both sides with $P^e(\tau, t_k') \exp\{-\int_{t_k'}^{\tau}[\rho+q+1/(x-t_{k-1}^*)]dx\}$ we recognize that

$$\begin{aligned}
&\dot{\tilde{\lambda}}_S^e(\tau)\, P^e\left(\tau, t_k'\right) e^{-\int_{t_k'}^{\tau}[\rho+q+1/(x-t_{k-1}^*)]dx} \\
&\qquad - \left(p^e(\tau) + \rho + q + \frac{1}{\tau - t_{k-1}^*}\right) \tilde{\lambda}_S^e(\tau)\, P^e\left(\tau, t_k'\right) e^{-\int_{t_k'}^{\tau}[\rho+q+1/(x-t_{k-1}^*)]dx} \\
&\qquad\qquad = -q \frac{\partial \pi_k^e\left[\bar{S}^e\left(\tau, t_{k-1}^*\right)\right]}{\partial \bar{S}^e\left(\tau, t_{k-1}^*\right)} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] P^e\left(\tau, t_k'\right) e^{-\int_{t_k'}^{\tau}[\rho+q+1/(x-t_{k-1}^*)]dx}
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad & \frac{\partial}{\partial \tau}\left(\tilde{\lambda}_S^e(\tau)\, P^e\left(\tau, t_k'\right) e^{-\int_{t_k'}^{\tau}[\rho+q+1/(x-t_{k-1}^*)]dx}\right) \\
&\qquad = -q \frac{\partial \pi_k^e\left[\bar{S}^e\left(\tau, t_{k-1}^*\right)\right]}{\partial \bar{S}^e\left(\tau, t_{k-1}^*\right)} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] P^e\left(\tau, t_k'\right) e^{-\int_{t_k'}^{\tau}[\rho+q+1/(x-t_{k-1}^*)]dx}.
\end{aligned}$$

Integrating the last result from τ to $\bar{t}_k^*$ we see that

$$\begin{aligned}
&\tilde{\lambda}_S^e\left(\bar{t}_k^*\right) P^e\left(\bar{t}_k^*, t_k'\right) e^{-\int_{t_k'}^{\bar{t}_k^*}[\rho+q+1/(x-t_{k-1}^*)]dx} - \tilde{\lambda}_S^e(\tau)\, P^e\left(\tau, t_k'\right) e^{-\int_{t_k'}^{\tau}[\rho+q+1/(x-t_{k-1}^*)]dx} \\
&\qquad = -\int_{\tau}^{\bar{t}_k^*} q \frac{\partial \pi_k^e\left[\bar{S}^e\left(x, t_{k-1}^*\right)\right]}{\partial \bar{S}^e\left(x, t_{k-1}^*\right)} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] P^e\left(x, t_k'\right) e^{-\int_{t_k'}^{x}[\rho+q+1/(z-t_{k-1}^*)]dz} dx.
\end{aligned}$$

Applying the transversality condition

$$\tilde{\lambda}_S^e\left(\bar{t}_k^*\right) = \frac{\partial \mathbb{U}_{k,2}^e\left(\bar{t}_k^*\right)}{\partial \bar{S}^e\left(\bar{t}_k^*, t_{k-1}^*\right)} = \frac{\partial \pi_k^e\left[\bar{S}^e\left(\bar{t}_k^*, t_{k-1}^*\right)\right]}{\partial \bar{S}^e\left(\bar{t}_k^*, t_{k-1}^*\right)} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right]$$

and rearranging further shows that

$$\begin{aligned}
\tilde{\lambda}_S^e(\tau) = &\int_{\tau}^{\bar{t}_k^*} q \frac{\partial \pi_k^e\left[\bar{S}^e\left(x, t_{k-1}^*\right)\right]}{\partial \bar{S}^e\left(x, t_{k-1}^*\right)} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] P^e(x, \tau)\, e^{-\int_{\tau}^{x}[\rho+q+1/(z-t_{k-1}^*)]dz} dx \\
&+ \frac{\partial \pi_k^e\left[\bar{S}^e\left(\bar{t}_k^*, t_{k-1}^*\right)\right]}{\partial \bar{S}^e\left(\bar{t}_k^*, t_{k-1}^*\right)} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] P^e\left(\bar{t}_k^*, \tau\right) e^{-\int_{\tau}^{\bar{t}_k^*}[\rho+q+1/(x-t_{k-1}^*)]dx}
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad \frac{\tilde{\lambda}_S^e(\tau)}{\tau - t_{k-1}^*} = &\int_{\tau}^{\bar{t}_k^*} q \frac{\left(\pi_k^e\left[\bar{S}^e\left(x, t_{k-1}^*\right)\right]\right)'}{x - t_{k-1}^*} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] P^e(x, \tau)\, e^{-[\rho+q](x-\tau)} dx \\
&+ \frac{\left(\pi_k^e\left[\bar{S}^e\left(\bar{t}_k^*, t_{k-1}^*\right)\right]\right)'}{\bar{t}_k^* - t_{k-1}^*} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] P^e\left(\bar{t}_k^*, \tau\right) e^{-[\rho+q](\bar{t}_k^*-\tau)}.
\end{aligned}$$

Inserting this result, together with (B-12), into (B-11) provides us with F.O.C. for search effort with costate variable substituted out

$$\begin{aligned}
c'\left[s^e(\tau)\right] = &\frac{1}{\rho+\delta} \int_{w_r^e(\tau)}^{\infty} \left[w - w_r^e(\tau)\right] dF(w) \\
&+ \int_{\tau}^{\bar{t}_k^*} q \frac{\left(\pi_k^e\left[\bar{S}^e\left(x, t_{k-1}^*\right)\right]\right)'}{x - t_{k-1}^*} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] P^e(x, \tau)\, e^{-[\rho+q](x-\tau)} dx \\
&+ \frac{\left(\pi_k^e\left[\bar{S}^e\left(\bar{t}_k^*, t_{k-1}^*\right)\right]\right)'}{\bar{t}_k^* - t_{k-1}^*} \left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] P^e\left(\bar{t}_k^*, \tau\right) e^{-[\rho+q](\bar{t}_k^*-\tau)}. \qquad \text{(B-13)}
\end{aligned}$$

B.2.2 Evolution over time and endpoint conditions

Differentiating equations (B-12) and (B-13) with respect to time we get the optimal paths of the control variables during the interview delay. Optimal path for the reservation wage reads

$$\dot{w}_r^e(\tau) = \rho \dot{U}_{k,2}^e(\tau) - \delta \left[\dot{U}_{k,2}^1(\tau) - \dot{U}_{k,2}^e(\tau)\right]. \tag{B-14}$$

Optimal path for search effort is likewise the time-derivative

$$\begin{aligned}
c''[s^e(\tau)]\,\dot{s}^e(\tau) = {} & \frac{1}{\rho+\delta}\int_{w_r^e(\tau)}^{\infty} \left[-\dot{w}_r^e(\tau)\right] dF(w) - q\frac{\left(\pi_k^e\left[\bar{S}^e\left(\tau, t_{k-1}^*\right)\right]\right)'}{\tau - t_{k-1}^*}\left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] \\
& + \int_{\tau}^{\bar{t}_k^*} q\frac{\left(\pi_k^e\left[\bar{S}^e\left(x, t_{k-1}^*\right)\right]\right)'}{x - t_{k-1}^*}\left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right]\frac{\partial}{\partial\tau}\left(P^e(x,\tau)\,e^{-[\rho+q](x-\tau)}\right)dx \\
& + \frac{\left(\pi_k^e\left[\bar{S}^e\left(\bar{t}_k^*, t_{k-1}^*\right)\right]\right)'}{\bar{t}_k^* - t_{k-1}^*}\left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right]\frac{\partial}{\partial\tau}\left(P^e\left(\bar{t}_k^*,\tau\right)e^{-[\rho+q]\left(\bar{t}_k^*-\tau\right)}\right)
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad c''[s^e(\tau)]\,\dot{s}^e(\tau) = {} & -\frac{\dot{w}_r^e(\tau)\,\bar{F}\left(w_r^e(\tau)\right)}{\rho+\delta} - q\frac{\left(\pi_k^e\left[\bar{S}^e\left(\tau, t_{k-1}^*\right)\right]\right)'}{\tau - t_{k-1}^*}\left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] \\
& + \left(p^e(\tau)+\rho+q\right)\left[\int_{\tau}^{\bar{t}_k^*} q\frac{\left(\pi_k^e\left[\bar{S}^e\left(x, t_{k-1}^*\right)\right]\right)'}{x - t_{k-1}^*}\left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right]P^e(x,\tau)\,e^{-[\rho+q](x-\tau)}dx\right. \\
& \left. + \frac{\left(\pi_k^e\left[\bar{S}^e\left(\bar{t}_k^*, t_{k-1}^*\right)\right]\right)'}{\bar{t}_k^* - t_{k-1}^*}\left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right]P^e\left(\bar{t}_k^*,\tau\right)e^{-[\rho+q]\left(\bar{t}_k^*-\tau\right)}\right].
\end{aligned}$$

Once $e = 1$, all terms but the first drop out, since $\left(\pi_k^1\left[\bar{S}^e\left(\tau, t_{k-1}^*\right)\right]\right)' = 0\ \forall\tau$ in the delay interval. Otherwise, when $e = 0$, the last term in the square brackets can be substituted out using eq.(B-13). With this substitution, optimal path for search effort reads

$$\begin{aligned}
\dot{s}^e(\tau) = {} & -\frac{\dot{w}_r^e(\tau)\,\bar{F}\left(w_r^e(\tau)\right)}{(\rho+\delta)\,c''[s^e(\tau)]} - \frac{q}{c''[s^e(\tau)]}\frac{\left(\pi_k^e\left[\bar{S}^e\left(\tau, t_{k-1}^*\right)\right]\right)'}{\tau - t_{k-1}^*}\left[U_{k+1,1}^e\left(t_k^*\right) - U^+\right] \\
& + (1-e)\frac{p^e(\tau)+\rho+q}{c''[s^e(\tau)]}\left[c'[s^e(\tau)] - \frac{1}{\rho+\delta}\int_{w_r^e(\tau)}^{\infty}\left\{w - w_r^e(\tau)\right\}dF(w)\right].
\end{aligned} \tag{B-15}$$

Finally note that with $q \to 0$, i.e. when the end of the delay interval is deterministic meaning that delay interval becomes just another scheduled interval, the path in (B-15) reduces to the path in (B-7).

Like before, optimal paths for both control variables depend on the evolution of the value of unemployment. Evolution of the value of unemployment can be obtained by differentiating (B-9) at any $\tau \in [t_k', \bar{t}_k^*)$. We get

$$\begin{aligned}
\dot{U}_{k,2}^e(\tau) & = \int_{\tau}^{\bar{t}_k^*}\left[y_{j(k)}^e(x) + p^e(x)\,\bar{W}_k^e(x) + q\mathbb{U}_{k,2}^e(x)\right]\frac{\partial}{\partial\tau}\left(P^e(x,\tau)\,e^{-[\rho+q](x-\tau)}\right)dx \\
& \quad - \left[y_{j(k)}^e(\tau) + p^e(\tau)\,\bar{W}_k^e(\tau) + q\mathbb{U}_{k,2}^e(\tau)\right] + \mathbb{U}_{k,2}^e\left(\bar{t}_k^*\right)\frac{\partial}{\partial\tau}\left(P^e\left(\bar{t}_k^*,\tau\right)e^{-[\rho+q]\left(\bar{t}_k^*-\tau\right)}\right) \\
& = \left(p^e(\tau)+\rho+q\right)\left\{\int_{\tau}^{\bar{t}_k^*}\left[y_{j(k)}^e(x) + p^e(x)\,\bar{W}_k^e(x) + q\mathbb{U}_{k,2}^e(x)\right]P^e(x,\tau)\,e^{-[\rho+q](x-\tau)}dx\right. \\
& \quad \left. + \mathbb{U}_{k,2}^e\left(\bar{t}_k^*\right)P^e\left(\bar{t}_k^*,\tau\right)e^{-[\rho+q]\left(\bar{t}_k^*-\tau\right)}\right\} - \left[y_{j(k)}^e(\tau) + p^e(\tau)\,\bar{W}_k^e(\tau) + q\mathbb{U}_{k,2}^e(\tau)\right] \\
& = \left(p^e(\tau)+\rho+q\right)U_{k,2}^e(\tau) - \left[y_{j(k)}^e(\tau) + p^e(\tau)\,\bar{W}_k^e(\tau) + q\mathbb{U}_{k,2}^e(\tau)\right] \\
& = (\rho+q)\,U_{k,2}^e(\tau) - y_{j(k)}^e(\tau) - p^e(\tau)\left[\bar{W}_k^e(\tau) - U_{k,2}^e(\tau)\right] - q\mathbb{U}_{k,2}^e(\tau).
\end{aligned}$$

Inserting into the last expression eq.(B-12), which tells us that $U_{k,2}^{e}(\tau) = \frac{1}{\rho+\delta}\left[w_r^e(\tau) + \delta U_{k,2}^{1}(\tau)\right]$, and the expression for $\bar{W}_k^e(\tau)$ we finally get

$$\dot{U}_{k,2}^{e}(\tau) = \frac{\rho+q}{\rho+\delta} w_r^e(\tau) - b_{j(k)} - v + c\left[s^e(\tau)\right] - \frac{s^e(\tau)}{\rho+\delta}\int_{w_r^e(\tau)}^{\infty}\left[w - w_r^e(\tau)\right] dF(w) + \delta\frac{\rho+q}{\rho+\delta} U_{k,2}^{1}(\tau) - q\mathbb{U}_{k,2}^{e}(\tau). \qquad \text{(B-16)}$$

Differential equations (B-14), (B-15) and (B-16) form a system that describes evolution of the optimal controls and the lifetime utility in the delay interval. This system can be solved backwards from the endpoint conditions for all the paths at $\bar{t}_k^*$. Endpoint conditions are found by solving the system of F.O.C. (B-12)-(B-13) together with eq.(B-9), all evaluated at $\bar{t}_k^*$

$$\begin{cases} w_r^e(\bar{t}_k^*) = \rho U_{k,2}^e(\bar{t}_k^*) - \delta\left[U_{k,2}^1(\bar{t}_k^*) - U_{k,2}^e(\bar{t}_k^*)\right] \\ c'\left[s^e(\bar{t}_k^*)\right] = \frac{1}{\rho+\delta}\int_{w_r^e(\bar{t}_k^*)}^{\infty}\left[w - w_r^e(\bar{t}_k^*)\right] dF(w) + \frac{\left(\pi_k^e\left[\bar{S}^e(\bar{t}_k^*, t_{k-1}^*)\right]\right)'}{\bar{t}_k^* - t_{k-1}^*}\left[U_{k+1,1}^e(t_k^*) - U^+\right] \\ U_{k,2}^e(\bar{t}_k^*) = \pi_k^e\left[\bar{S}^e(\bar{t}_k^*, t_{k-1}^*)\right] U_{k+1,1}^e(t_k^*) + \left(1 - \pi_k^e\left[\bar{S}^e(\bar{t}_k^*, t_{k-1}^*)\right]\right) U^+ \end{cases}$$

for $\left\{w_r^e(\bar{t}_k^*), s^e(\bar{t}_k^*), U_{k,2}^e(\bar{t}_k^*)\right\}$, where for after the last interview ($k = 3$) we know that $U_{4,1}^e(t_k^*) \equiv U^-$.

Solution algorithm for optimal paths derived above is described in detail in Appendix C, Steps 1.1 to 1.5, of the main text.

C Welfare Indicators for the Cost-Benefit Analysis

For computational considerations we take welfare indicators conditional on randomly drawn durations of interview delays, rather then explicitly model their expected values with respect to the distribution of random delay times. Hence, below t_1 designates a realization of a random draw. Note also that we only write down the expressions for the first subperiod, $[t_0, t_1]$. The expressions for the next three subperiods are analogous and are therefore not reported explicitly.

1. Expected discounted stream benefits and leisure net of search costs plus future labor earnings to unemployed. This measure is nothing but the lifetime value of unemployment for $e = 0$ (with expectations being taken at the moment of notification t_0).

$$\begin{aligned} U_1^0(t_0) &= \int_{t_0}^{t_1'}\left[\left(b + v - c\left[s^0(x)\right]\right) + p\left(w_r^0(x), s^0(x)\right)\bar{W}_1^0(x)\right] P^0(x, t_0)\, e^{-\rho(x-t_0)} dx \\ &\quad + P^0(t_1', t_0)\, e^{-\rho(t_1'-t_0)}\left\{\int_{t_1'}^{t_1}\left[\left(b + v - c\left[s^0(x)\right]\right) + p\left(w_r^0(x), s^0(x)\right)\bar{W}_1^0(x) + q\mathbb{U}_1^0(x)\right]\right. \\ &\quad \left.\times P^0(x, t_1')\, e^{-[\rho+q](x-t_1')} dx\right\} + P^0(t_1, t_0)\, e^{-\rho(t_1-t_0)-q(t_1-t_1')}\mathbb{U}_1^0(t_1) \end{aligned} \qquad \text{(B-17)}$$

where $\mathbb{U}_1^0(x) = \pi_1^0\left[\bar{S}^0(x, t_0)\right] U_2^0(t_1) + \left(1 - \pi_1^0\left[\bar{S}^0(x, t_0)\right]\right) U^+$.

2. Expected discounted stream of labor earnings. This measure excludes instantaneous utility of unemployment (benefits and leisure net of search costs) from the lifetime value of unemployment:

$$\begin{aligned} X_1^0(t_0) &= \int_{t_0}^{t_1'}\left[(0 + 0 - 0) + p\left(w_r^0(x), s^0(x)\right)\bar{W}_{X_1}^0(x)\right] P^0(x, t_0)\, e^{-\rho(x-t_0)} dx \\ &\quad + P^0(t_1', t_0)\, e^{-\rho(t_1'-t_0)}\left\{\int_{t_1'}^{t_1}\left[(0 + 0 - 0) + p\left(w_r^0(x), s^0(x)\right)\bar{W}_{X_1}^0(x) + q\mathbb{X}_1^0(x)\right]\right. \\ &\quad \left.\times P^0(x, t_1')\, e^{-[\rho+q](x-t_1')} dx\right\} + P^0(t_1, t_0)\, e^{-\rho(t_1-t_0)-q(t_1-t_1')}\mathbb{X}_1^0(t_1) \end{aligned} \qquad \text{(B-18)}$$

where

$$\begin{aligned}
\bar{W}_{X_1}^0(x) &= \int_{w_r(x)}^{\infty} \frac{1}{\rho+\delta}\left[w+\delta X_1^1(x)\right]\frac{f(w)}{\bar{F}(w_r(x))}dw. \\
\mathbb{X}_1^0(x) &= \pi_1^0\left[\bar{S}^0(x,t_0)\right]X_2^0(t_1)+\left(1-\pi_1^0\left[\bar{S}^0(x,t_0)\right]\right)X^+,
\end{aligned}$$

X^+ having the same interpretation as U^+ above.

3. Expected discounted expenditures of public authorities.

$$\begin{aligned}
Y_1^0(t_0) &= \int_{t_0}^{t_1'}\left[(b+0-0)-p\left(w_r^0(x),s^0(x)\right)\bar{W}_{Y_1}^0(x)\right]P^0(x,t_0)e^{-\rho(x-t_0)}dx \\
&\quad +P^0\left(t_1',t_0\right)e^{-\rho(t_1'-t_0)}\left\{\int_{t_1'}^{t_1}\left[(b+0-0)-p\left(w_r^0(x),s^0(x)\right)\bar{W}_{Y_1}^0(x)+q\mathbb{Y}_1^0(x)\right]\right. \\
&\quad \left.\times P^0\left(x,t_1'\right)e^{-[\rho+q](x-t_1')}dx\right\}+P^0(t_1,t_0)e^{-\rho(t_1-t_0)-q(t_1-t_1')}\mathbb{Y}_1^0(t_1) \\
&\quad +\tilde{c}P^0(t_1,t_0)e^{-\rho(t_1-t_0)-q(t_1-t_1')} \qquad \text{(B-19)}
\end{aligned}$$

where $\tilde{c}$ is a per capita cost of interview and

$$\begin{aligned}
\bar{W}_{Y_1}^0(x) &= \int_{w_r(x)}^{\infty}\frac{1}{\rho+\delta}\left[\left[\tilde{w}(w)-w\right]+\delta Y_1^1(x)\right]\frac{f(w)}{\bar{F}(w_r(x))}dw \\
\mathbb{Y}_1^0(x) &= \pi_1^0\left[\bar{S}^0(x,t_0)\right]Y_2^0(t_1)+\left(1-\pi_1^0\left[\bar{S}^0(x,t_0)\right]\right)Y^+, \\
\tilde{w}(w) &= w+T(w)\approx w+\left[\alpha_0+\alpha_1 w+\alpha_2 w^2\right]
\end{aligned}$$

with $\tilde{w}(w)$ denoting the labour cost of paying the net wage w, and $T(w)$ is the amount of taxes paid. Coefficients $\alpha_i, i\in\{0,1,2\}$, are estimated externally.

4. Expected discounted value of the net output.

$$\begin{aligned}
Z_1^0(t_0) &= \int_{t_0}^{t_1'}\left[\left(0+v-c\left[s^0(x)\right]\right)+p\left(w_r^0(x),s^0(x)\right)\bar{W}_{Z_1}^0(x)\right]P^0(x,t_0)e^{-\rho(x-t_0)}dx \\
&\quad +P^0\left(t_1',t_0\right)e^{-\rho(t_1'-t_0)}\left\{\int_{t_1'}^{t_1}\left[\left(0+v-c\left[s^0(x)\right]\right)+p\left(w_r^0(x),s^0(x)\right)\bar{W}_{Z_1}^0(x)+q\mathbb{Z}_1^0(x)\right]\right. \\
&\quad \left.\times P^0\left(x,t_1'\right)e^{-[\rho+q](x-t_1')}dx\right\}+P^0(t_1,t_0)e^{-\rho(t_1-t_0)-q(t_1-t_1')}\mathbb{Z}_1^0(t_1) \qquad \text{(B-20)}
\end{aligned}$$

where

$$\begin{aligned}
\bar{W}_{Z_1}^0(x) &= \int_{w_r(x)}^{\infty}\frac{1}{\rho+\delta}\left[\tilde{w}(w)+\delta Z_1^1(x)\right]\frac{f(w)}{\bar{F}(w_r(x))}dw \\
\mathbb{Z}_1^0(x) &= \pi_1^0\left[\bar{S}^0(x,t_0)\right]Z_2^0(t_1)+\left(1-\pi_1^0\left[\bar{S}^0(x,t_0)\right]\right)Z^+.
\end{aligned}$$